12.1 Probability of an Event

- An experiment is a procedure that results in one out of a number of possible outcomes. The set of all possible outcomes is called the sample space of the experiment. A subset of the sample space is called an event.

- Discrete probability is concerned with experiments in which the sample space is a finite or countably infinite set.

- A probability distribution over the outcomes of an experiment with a countable sample space S is a function p from S to the set of real numbers in the interval from 0 to 1 with the property that:

$$\sum_{s \in S} p(s) = 1$$

- The probability of outcome s is $p(s)$. If $E \subseteq S$ is an event, then the probability of event E is:

$$p(E) = \sum_{s \in E} p(s)$$

- The probability distribution in which every outcome has the same probability is called the uniform distribution.

12.2 Unions and Complements of Events

- Two events are <u>mutually exclusive</u> if the two events are disjoint (that is, the intersection of the two events is empty).

- If E_1 and E_2 are mutually exclusive:

$$p(E_1 \cup E_2) = p(E_1) + p(E_2)$$

- If E_1 and E_2 are <u>not</u> mutually exclusive:

$$p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$$

- <u>Complements</u>:

$$p(\overline{E}) = 1 - p(E)$$

12.3 Conditional Probability and Indendence

- The <u>conditional probability</u> of E given F is $p(E|F) = \dfrac{p(E \cap F)}{p(F)}$

- If the distribution is uniform, then the conditional probability becomes:

$$p(E|F) = \frac{p(E \cap F)}{p(F)} = \frac{\left(\frac{|E \cap F|}{|S|}\right)}{\left(\frac{|F|}{|S|}\right)} = \frac{|E \cap F|}{|F|}$$

- <u>Complement and conditional Probability</u>
  - If E and F are both events in the same sample space S, then the probability of E and the probability of $\overline{E}$ still sum to 1, even when conditioned on the event F.

$$p(E|F) + p(\overline{E}|F) = 1$$

- Two events are <u>independent</u> if conditioning on one event does not change the probability of the other event.

Definition: Independent Events

Let E and F be two events in the same sample space. The following 3 conditions are equivalent:

1. $p(E|F) = \dfrac{p(E \cap F)}{p(F)} = p(E)$

2. $p(E \cap F) = p(E) \cdot p(F)$

3. $p(F|E) = \dfrac{p(E \cap F)}{p(E)} = p(F)$

- If one of the 3 conditions hold, then events E and F are independent.

- If X and Y are events in the same sample space, and X and Y are independent, then:

$$p(X \cap Y) = p(X) \cdot p(Y)$$

- Events $A_1, \ldots, A_n$ in sample space S are <u>mutually independent</u> if the probability of the intersection of any subset of the events is equal to the product of the events in the subset. In particular, if $A_1, \ldots, A_n$ are mutually independent, then:

$$p(A_1 \cap A_2 \cap \ldots \cap A_n) = p(A_1) \cdot p(A_2) \cdot \ldots \cdot p(A_n)$$